Topics in Geometry

The following problems will test your ability to use these fundamental topics covered in geometry: Similarity, Right Triangles, and Trigonometry. These problems may require concepts in higher math classes. Once again, you should notice that many of these problems are word problems.

1. In triangle ABC, $AC = 4$, $BC = 3$, $AB = 5$, and $\angle ACB = 90^\circ$. The infinite sequence of points $C_1, C_2, C_3, C_4, \ldots$ is generated in the following manner: $C_1$ is the foot of the altitude from $C$ to the side $\overline{AC}$, $C_2$ is the foot of the altitude from $C_1$ to the side $\overline{AC}$, $C_3$ is the foot of the altitude from $C_2$ to side $\overline{AB}$, and so on. Evaluate the expression: (Note: $C = C_0$)

$$\sum_{n=1}^{\infty} C_{n-1} \cdot C_n = C_0C_1 + C_1C_2 + C_2C_3 + C_3C_4 + \cdots$$

2. In rectangle ABCD points F and G lie on segment AB so that $AF = FG = GB$ and E is the midpoint of segment DC. Also, segment AC intersects segment EF at H and segment EG at J. The area of rectangle ABCD is 70 square units. What is the area of triangle AHF?
3. In triangle ABC, $AB = 6$ and $BC = 8$. Find $AC$ in each of the following cases (exact solutions only!!)
   A. $\angle B = 30^\circ$
   B. $\angle B = 45^\circ$
   C. $\angle B = 135^\circ$

4. The hypotenuse of a right triangle has length 8. The area of the triangle is also 8 units squared. What is the perimeter of the triangle?

5. At the beginning a 50 ft. ladder is leaning against a wall. As you start to climb the ladder, the ladder begins to slides down until it stops at a point 8 feet below where it originally started. During the slide, the base of the ladder slide 16 feet from its original position. How far is the top of the ladder from the ground after the slide given that the wall is perpendicular to the ground?

6. You are given the following information about triangle ABC. $AB = AC$, $BC = \sqrt{3} - 1$, and $\angle BAC = 30^\circ$. What is the length of $AB$?

7. A surveyor measures the angle of elevation from her feet to the top of a building to be $5^\circ$. The surveyor knows that the building is 500 feet tall. Assuming that the ground is perfectly flat and level between the surveyor and the building, how far away is the building?

8. You are standing at the peak of a mountain that is 14000 feet above sea level. The angle of depression from this peak to a nearby smaller peak is $4^\circ$. On your map, these two peaks are represented by points that are 1 inch apart. If each inch on your map represents 1.2 miles, how many feet above sea level is the second peak?

9. A pirate is on a hill looking at a building. The building is 400 feet tall. The angle of elevation from the pirate to the building is $4^\circ$ and the angle of depression from the pirate to the bottom of the building is $2^\circ$. What is the shortest distance the pirate will need to travel to reach the building? (Introduction to Geometry, pg.502 18.29)

10. If $\sin x = 3 \cos x$, then what is $\sin x \cdot \cos x$?