Topics in Precalculus – Advanced Mathematics

The following problems will focus on the depth of understanding in the following topics: Functions, Trigonometric Functions & Identities and their applications. These questions may require you to intertwine multiple mathematical disciplines. Remember that that these are advanced mathematical problems which will require all of the mathematics you have learned.

1. Suppose a function \( f(x) \) has domain \((−1, 1)\). Find the domains of the following functions:
   A. \( f(x + 1) \)
   B. \( f\left(\frac{1}{x}\right) \)
   C. \( f(\sqrt{x}) \)
   D. \( f\left(\frac{x+1}{x-1}\right) \)

2. Find the domain and range (algebraically) of the function \( f(x) = \sqrt{2 - x - x^2} \)

3. Suppose \( f(f(x)) = x \) for all \( x \) for which \( f(x) \) is defined and \( f(0) = 2003 \). Find all the roots of the equation \( f(x) = 0 \).

4. Suppose \( f(t^2 + 1) = t^4 + 5t^2 + 3 \). What is \( f(t^2 - 1) \)?

5. What is the period of \( f(x) = \sin x \cos x \)?

6. How many values of \( x \) satisfy the equation: \( \sin x = \frac{x^2}{625\pi^2} \)?

7. Evaluate: \( \arccos\left(\sin\left(\frac{6\pi}{7}\right)\right) \)

8. Solve: \( \sin(3\theta) = \frac{1}{2} \)

9. While evaluating the sine of a particular angle, an absent minded student failed to notice that his calculator was not in the correct angular mode. He was very lucky to end up with the right answer. What are the two smallest possible values of \( x \) such that the sine of \( x \) degrees equals the sine of \( x \) radians?

10. What is the value of \( \tan x \) given that \( \sin x + \cos x = \frac{1}{5} \) and \( \frac{\pi}{2} < x < \pi \)?
11. Solve, algebraically or with justification (no graphing calculator solutions allowed), the following equation: \( \tan 7x - \sin 6x = \cos 4x - \cot 7x \)

12. Algebraically find the value of: \( 10 \cot (\cot^{-1} 3 + \cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21) \)

13. Express the triple angle formula of tangent (i.e. \( \tan(3x) \)) in terms of \( \tan x \).

14. Suppose \( x \) is an angle in the second quadrant such that \( \cos x = -\frac{8}{17} \), what is the value of \( \tan \left( \frac{3x}{2} \right) \)?

15. What is the value of \( \sin 2x \) given that \( \sin x + \cos x + \tan x + \cot x + \sec x + \csc x = 7 \)?

16. Let ABCD be a quadrilateral with the following properties. \( \angle A = \angle C, AB = CD = 180 \), and \( AD \neq BC \). The perimeter of the quadrilateral is 640. What is the EXACT value of \( \cos A \)?

17. You are given triangle ABC. \( AB = 13, BC = 15, \) and \( CA = 14 \). Point D is on segment BC and \( CD = 6 \). Point E is on segment BC such that \( CE > CD \) and \( \angle BAE = \angle CAD \). What is the length of BE?